Table 10.3: Necessary equations.

B_n = contribution of LSB = $V_{max}/(2^n - 1)$	
B_i = contribution of bit $i = FSR/2^i = 2^{n-i}B_n$	
FSR – full scale range	*electrical span* – $FSR - 1\ LSB$ (since FSR is the one bit beyond the MSB)
e_i = bit error = $V_i - B_i$	
$(+)$ – positive error set	$(-)$ – negative error set

To reduce noise, we average a number of readings at the voltmeter (provided that the noise effects on samples are independent.) We use an 8000 s/s sampling rate, and average 100 samples. Digital crosstalk is eliminated by silencing the digital bus for 1 ms before operating the voltmeter.

Derivation of INL Measure. The method measures $\Delta_i = \Uparrow \left\{ \begin{smallmatrix} 0001000000 \\ 0000111111 \end{smallmatrix} \right\}$, the analog voltage change caused by one code increment in bit i, where 1 (MSB) $\leq i \leq n$ (LSB). From all measured Δs, we reconstruct the DAC point map. Error is the difference between the actual bit contribution and the nominal value. $V_{max} = \sum V_i = \sum B_i$ so $\sum e_i = 0$ meaning that errors have both positive and negative signs and sum to 0. When positive and negative errors are grouped separately, $\sum(+) + \sum(-) = 0$. The two sums define the limits of *integral non-linearity* (INL), and are averaged to limit the effect of measurement error:

$$INL = \frac{\sum(+) - \sum(-)}{2B_n} \tag{10.5}$$

So, INL comes from the difference of sets V_i and B_i, which come from the increment voltage set Δ_i measured at the test fixture.

The algorithm removes non-noise errors:

- The large denominator $2^n - 1$ is twice the size of the positive coefficients and twice the negative coefficient sum. An error of 1% in a Δ_i value causes only 1/2% error in DNL.

- The sum of coefficients in the numerator is zero, so if all Δ_is have the same constant error, it vanishes in e_i.

- In Equation 10.5, if all Δ_i change by the same proportional error, the numerator and denominator change by the same proportion, so the INL is still correct (this eliminates voltmeter and amplifier gain error during measurement.) The remaining errors are random noise and amplifier/voltmeter tracking error.

V_i is obtained from summing the immediately preceding step, Δi, and binary multiples of all lower-order Δs [411]. The first n elements of the series:

$$1 + 1 + 2 + 4 + 8 + 16 + \ldots$$