Optical Flow - Testing of the Horn and Schunck Method

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Outline

- Motiviation
- Definition of Optical Flow
- Horn & Schunck Algorithm
- Experiments
- Conclusions

Research

- Computer Vision research under Dr. Hodel
- Goal → combine vision data with IMU data for navigation
- Determine orientation, velocity and location through vision
- Area of CV research → Optical Flow

What is optical flow?

- Definition: *“The distribution of apparent velocities of movement of brightness patterns in an image.”*[1]
- Involves projecting 3-D vision onto a 2-D plane
- Computes motion vectors of objects within an image sequence
- Vectors can be used to analyze an objects motion $\rightarrow$ navigation
- Interstate example

Horn and Schucnk Method

- 1981 - Horn and Schucnk publish *Determining Optical Flow*[1]
- One of the first optical flow methods
- Idea: use intensisty derivatives to estimate velocities

H&S Method - Assumptions

I. Object within image is a flat surface

II. Illumination on the image is constant and uniform

III. Reflectance varies smoothly, no spatial discontinuities

→ These assumptions assure that intensity is differentiable

Constraints

- Constrainted Brightness Equation
  - Intensity at a particular point within on object does not change over time:

$$
\begin{aligned}
\frac{dE}{dt} &= 0 \\
\frac{\partial E}{\partial x}\frac{dx}{dt} + \frac{\partial E}{\partial y}\frac{dy}{dt} + \frac{\partial E}{\partial t} &= 0 \\
E_x u + E_y v + E_t &= 0
\end{aligned}
$$

Constraints

- Constrained Brightness Eq. (cont.) Rearranging the equation:

$$\begin{bmatrix} E_x & Ey \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -E_t$$

Component of movement in the direction of the brightness gradient (E_x, E_y) is

$$-\frac{E_t}{\sqrt{E_x^2 + E_y^2}}$$

Aperture Effect

- Aperture Effect
- Barber shop pole example
- CBE $\Rightarrow$ motion can only be detected in the direction of the brightness gradient
- H&S method - subject to aperture problem

Constraints

- Smoothness Constraint
  - Neighboring points on a moving object have similar velcocities, therefore the brightness patterns in the image vary smoothly [1].
  - Occluding objects will give discontinuities in optical flow
  - Smoothness constraint adds difficulty in areas of occluding edges that are undergoing different movement

Smoothness Constraint (Cont.)

- Expressing the constraint - minimize the square of the gradient maginutde

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 \; and \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 .$$

- The smoothness of the optical flow field can also be measured by the Laplacians of u and v:

$$\begin{aligned} \nabla^2 u &= \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \\ \nabla^2 v &= \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \end{aligned}$$

Estimations-Derivatives

- E_x, E_y, and E_t must be estimated for the CBE
- Take an average over a 4x4x2 block of back to back images

$$
\begin{aligned}
Ex &\approx \frac{1}{4}[E_{ii+1,jj+1,kk} - E_{ii+1,jj,kk} + E_{ii,jj+1,kk} - E_{ii,jj,kk} \cdots \\
&\quad + E_{ii+1,jj+1,kk+1} - E_{ii+1,jj,kk+1} + E_{ii,jj+1,kk+1} - E_{ii,jj,kk+1}] \\
Ey &\approx \frac{1}{4}[E_{ii,jj,kk} - E_{ii+1,jj,kk} + E_{ii,jj+1,kk} - E_{ii+1,jj+1,kk} \cdots \\
&\quad + E_{ii,jj,kk+1} - E_{ii+1,jj,kk+1} + E_{ii,jj+1,kk+1} - E_{ii+1,jj+1,kk+1}] \\
Ey &\approx \frac{1}{4}[E_{ii+1,jj,kk+1} - E_{ii+1,jj,kk} + E_{ii,jj,kk+1} - E_{ii,jj,kk} \cdots \\
&\quad + E_{ii+1,jj+1,kk+1} - E_{ii+1,jj+1,kk} + E_{ii,jj+1,kk+1} - E_{ii,jj+1,kk}]
\end{aligned}
$$

Estimations-Laplacians

- Laplacian estimates

$$\nabla^2 u \approx k(\bar{u}_{i,j,k} - u_{i,j,k}) \;\; and \;\; \nabla^2 u \approx k(\bar{v}_{i,j,k} - v_{i,j,k})$$

where $\bar{u}$ and $\bar{v}$ are local averages of the velocity vectors

$$\begin{aligned}
\bar{u}_{i,j,k} &= \frac{1}{6}[u_{ii-1,jj} + u_{ii,jj+1} + u_{ii+1,jj} + u_{ii,jj-1}]\cdots \\
&\quad + \frac{1}{12}[u_{ii-1,jj-1} + u_{ii-1,jj+1} + u_{ii+1,jj+1} + u_{ii+1,jj-1}] \\
\bar{v}_{i,j,k} &= \frac{1}{6}[v_{ii-1,jj} + v_{ii,jj+1} + v_{ii+1,jj} + v_{ii,jj-1}]\cdots \\
&\quad + \frac{1}{12}[v_{ii-1,jj-1} + v_{ii-1,jj+1} + v_{ii+1,jj+1} + v_{ii+1,jj-1}]
\end{aligned}$$

Minimization

- Algorithm approach $\rightarrow$ minimize the error in brightness equation and the departure from smoothness.

$$\varepsilon_b = E_x u + E_y v + Et$$
$$\varepsilon_c^2 = \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2$$

The total error to be minimized is then:

$$\varepsilon^2 = \int\int (\alpha^2 \varepsilon_c^2 + \varepsilon_b^2) dxdy$$

- Constrained least squares minimization

Iterative Solution

- Direct solution to minimization → computationally expensive
- Iterative solution is developed
- Velocity estimate based on estimated derivatives and average of previous estimates

$$u^{n+1} = \overline{u}^n - Ex\left(\frac{Ex\overline{u}^n + Ey\overline{v}^n + Et}{\alpha^2 + E_x^2 + E_y^2}\right)$$

$$v^{n+1} = \overline{v}^n - Ex\left(\frac{Ex\overline{u}^n + Ey\overline{v}^n + Et}{\alpha^2 + E_x^2 + E_y^2}\right)$$

Algorithm

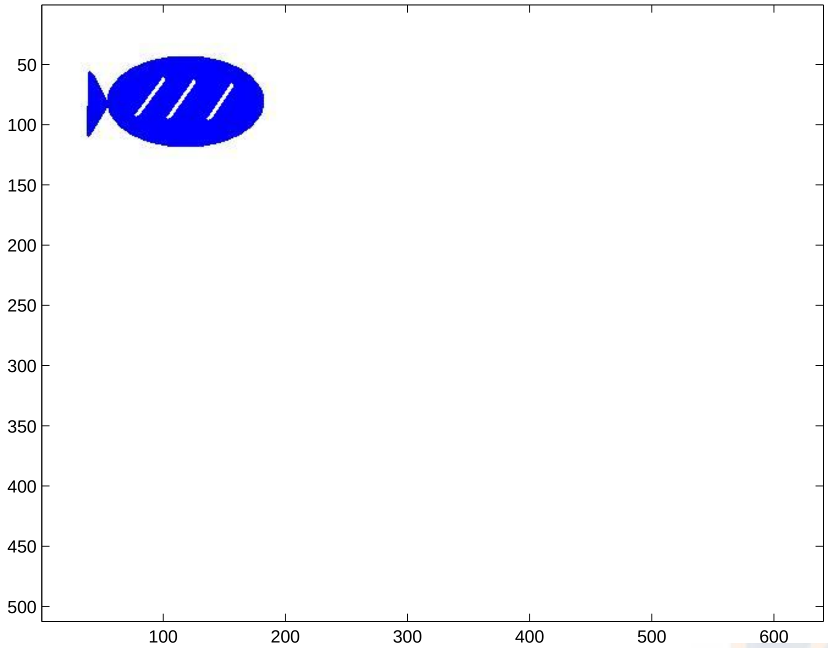
```
======Optical Flow Algorithm==============
obtain image sequence
get size of image sequence, M(x), N(y), number(t)
set alpha; num_its = number of images
for kk = 1:number
    for ii = 1:M
        for jj = 1:N
            compute Ex, Ey and Et
        end
    end
    for nn = 1:num_its
        for ii = 1:M
            for jj = 2:N
                compute ubar
                compute vbar
                update u
                update v
            end
        end
    end
end
```

Testing the H&S Algorithm

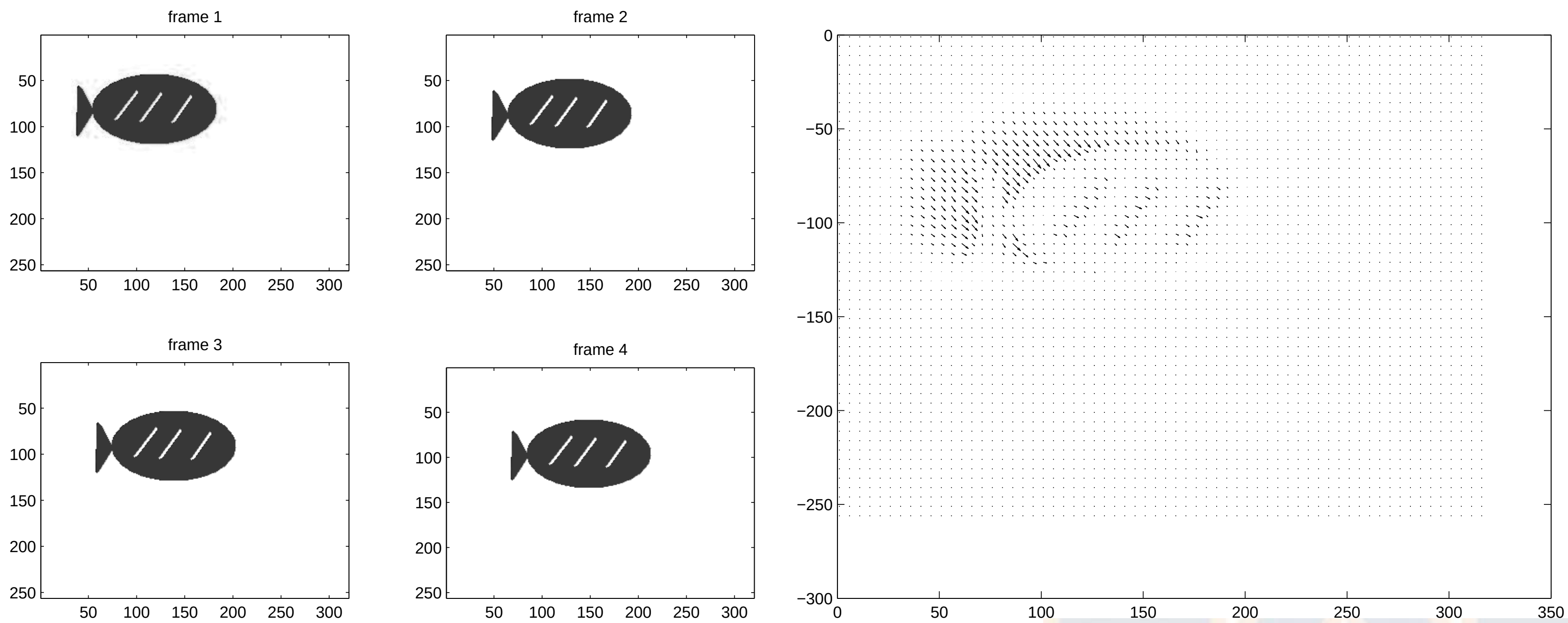
- Horn & Schucnk algorithm implemented in MATLAB
- Tested on:
  - Synthetic image sequence
  - Real image sequence
- $\alpha = 20$
- number of iterations = 32
- Computed velocities displayed with a quiver plot

Synthetic Image Sequence

- Original created in paint
- Sequence generated in MATLAB



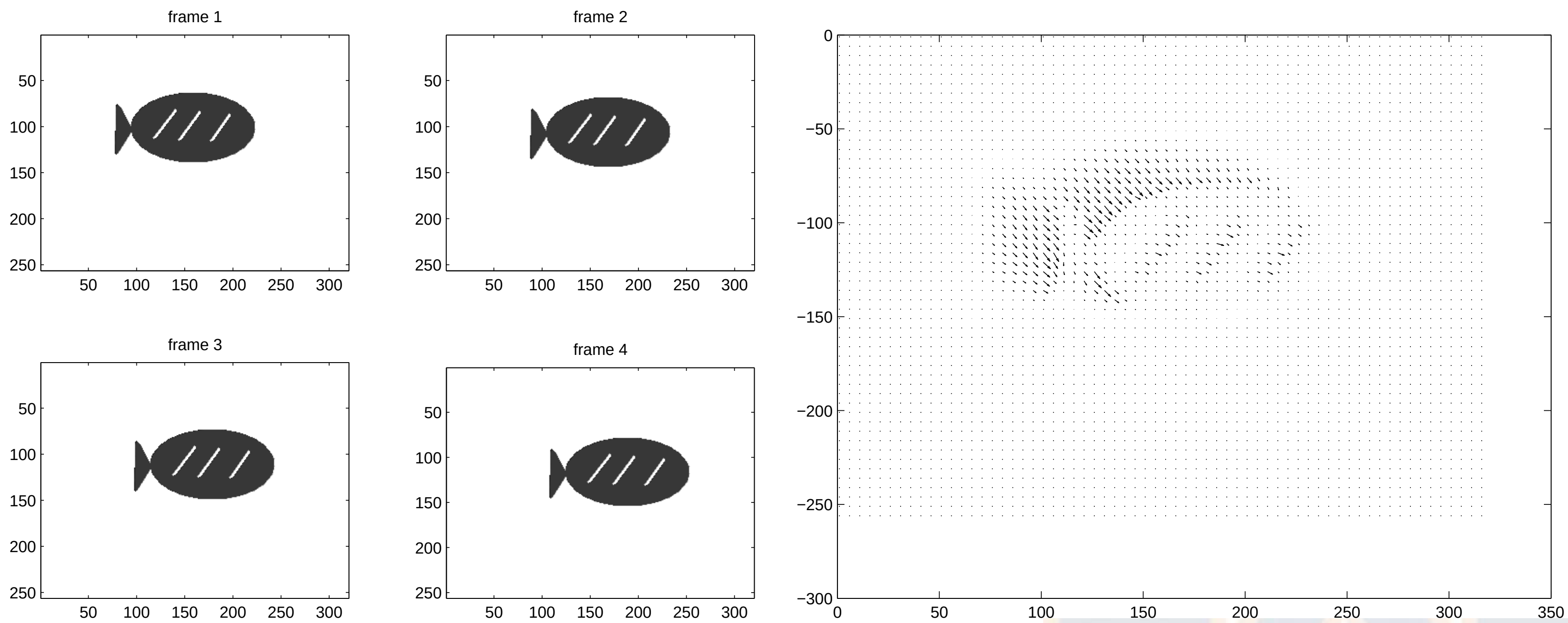
Frames 1-4



(a) Frames 1 - 4 (b) Optical Flow

Figure 1: First four frames of synthetic sequence and the computed optical flow

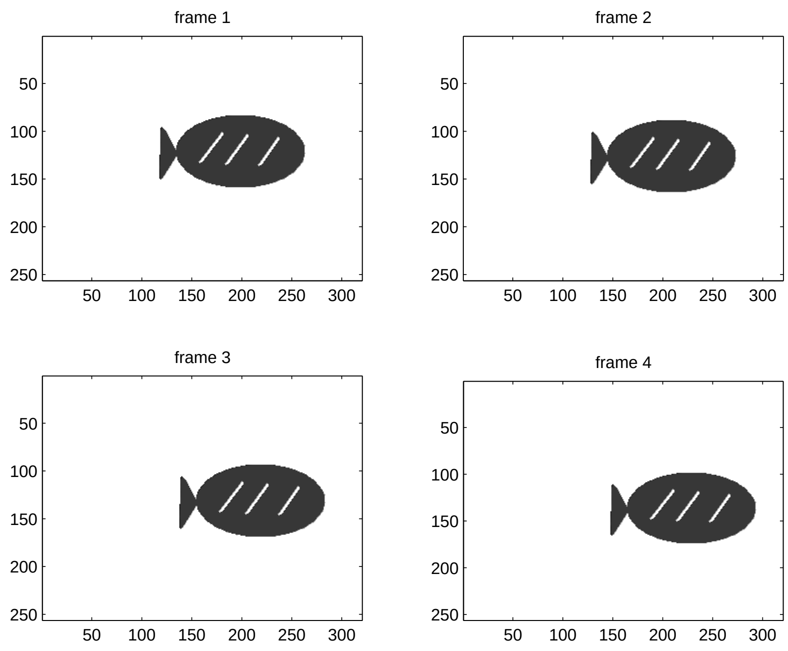
Frames 5-8



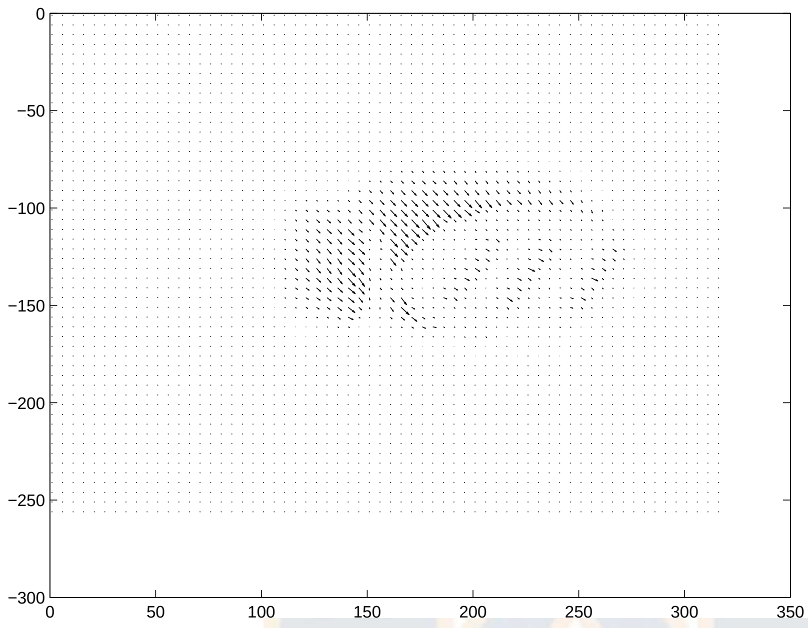
(a) Frames 5 - 8 (b) Optical Flow

Figure 2: Next four frames and computed optical flow

Frames 9-12



(a) Frames 9 - 12



(b) Optical Flow

Figure 3: Last four frames and computed optical flow

Real Image Sequence

- Sequence taken from a digital camera
- Images are of a balloon tied to a ceiling fan

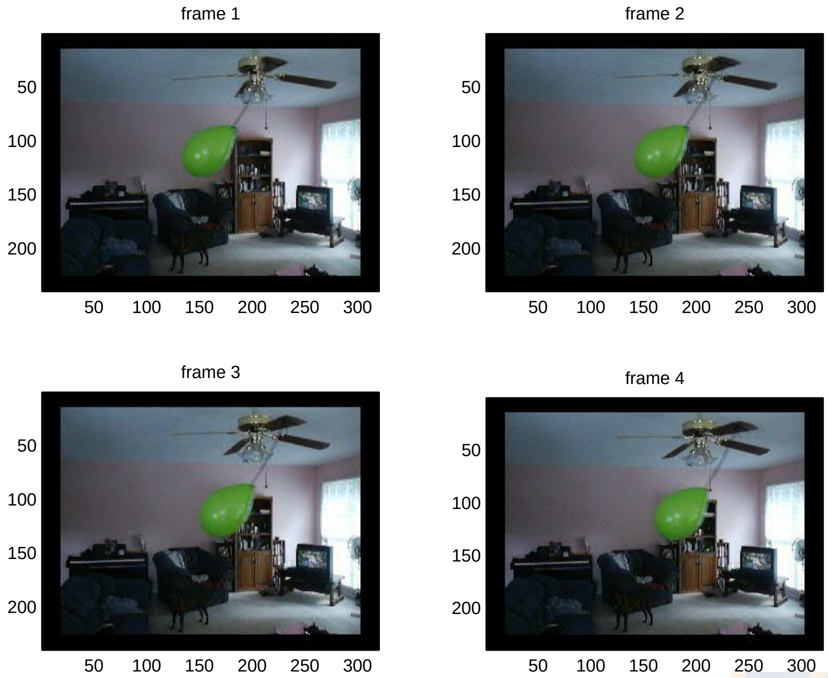
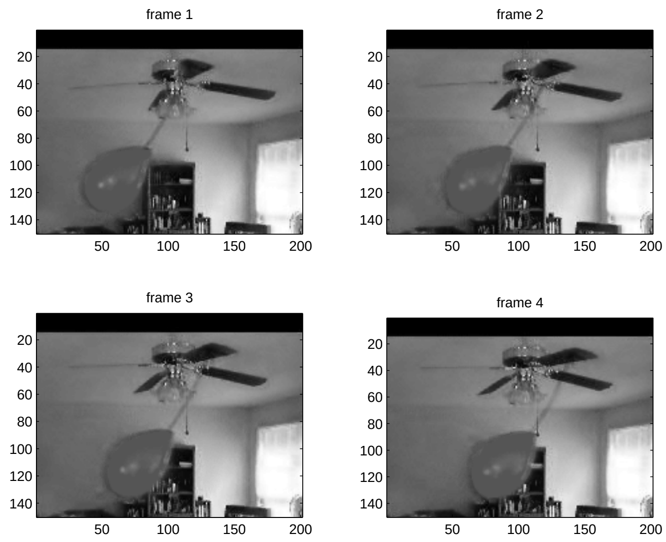
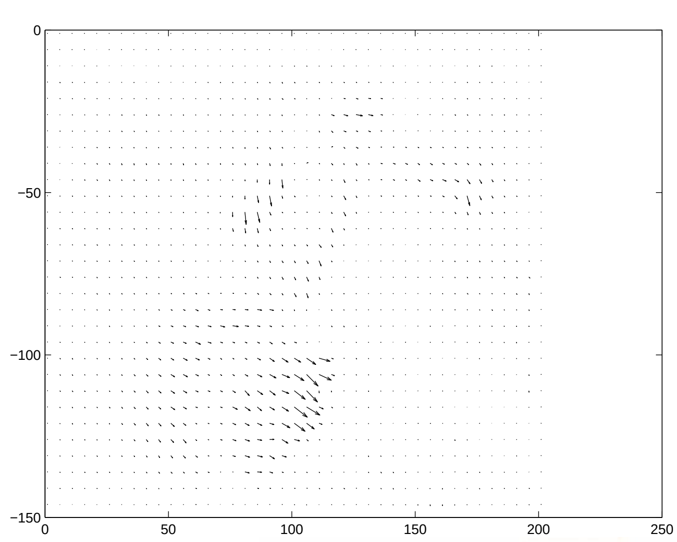


Figure 4: First four images in balloon sequence

Frames 1-4

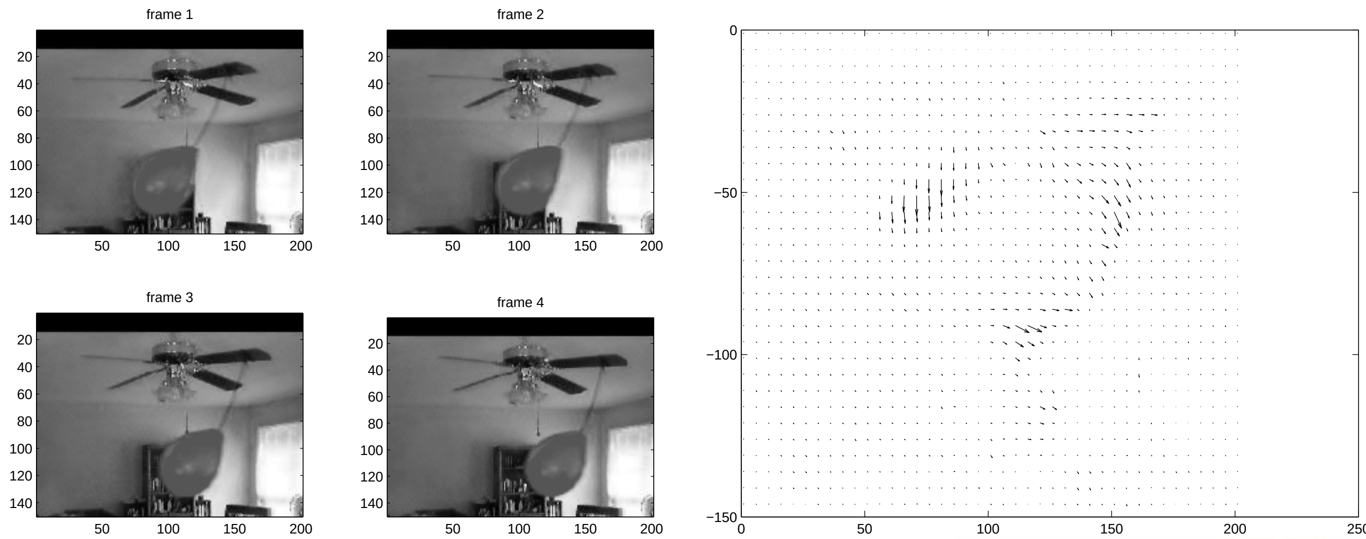




(a) Frames 1 - 4 (b) Optical Flow

Figure 5: First four frames of balloon sequence and the computed optical flow

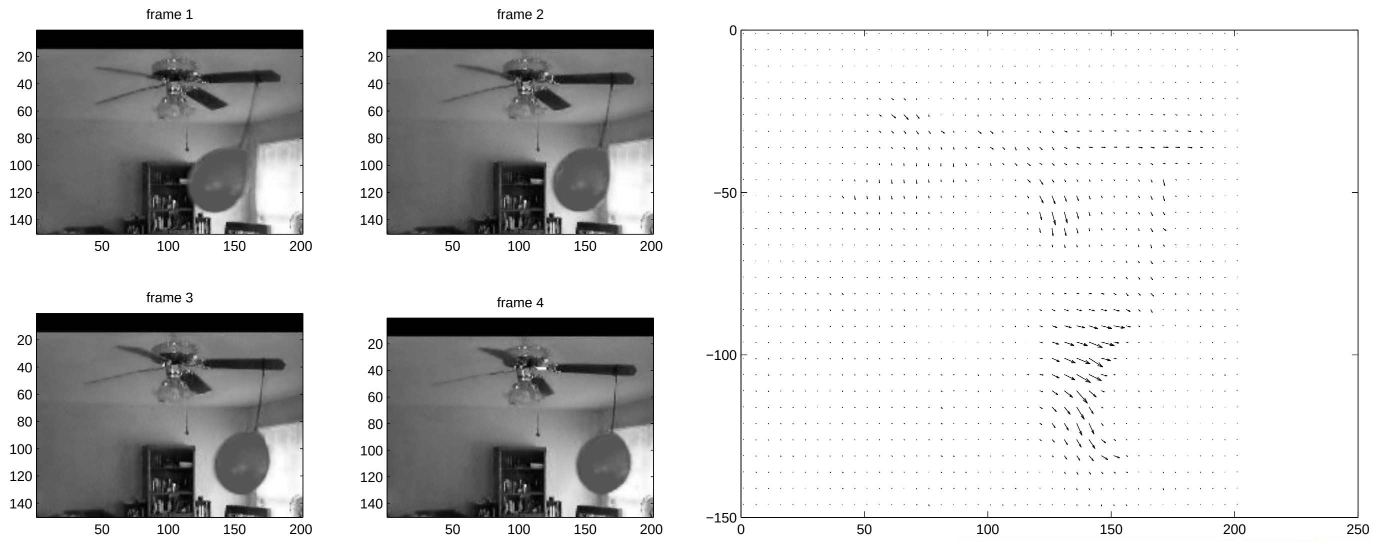
Frames 5-8



(a) Frames 5 - 8 (b) Optical Flow

Figure 6: Next four frames and computed optical flow

Frames 9-12



(a) Frames 9 - 12 (b) Optical Flow

Figure 7: Last four frames and computed optical flow

Animation

Start animation

Conclusions

- Strengths
  - H&S works well when conditions are suitable for constraints
  - Easy to implement
  - Methods are intuitive (What's different between two images?)
- Weaknesses
  - Subject to aperture effect
  - Conditions usually aren't suitable for using constraints
- Questions?

References

[1] B.K.P Horn and B.G. Schunk. Determining optical flow. *Artificial Intelligence*, 17:185–203, 1981.