

## ode45 - Differential Equation Solver

This routine uses a variable step Runge-Kutta Method to solve differential equations numerically. The syntax for **ode45** for first order differential equations and that for second order differential equations are basically the same. However, the .m files are quite different.

I. First Order Equations 
$$\begin{cases} y' = f(t, y) \\ y(t_0) = y_0 \end{cases}$$

A. Create a .m file for  $f(t, y)$  (see the tutorial on numerical methods and m files on how to do this). Save file as, for example, **yp.m** .

B. Basic syntax for **ode45** . At a MATLAB prompt type :

```
[t,y]=ode45('yp',[t0,tf],y0);  
(your version of ode45 may not require brackets around t0, tf)
```

$$\begin{cases} \text{yp} = \text{the .m file of the function } f(t, y) \text{ saved as yp.m} \\ \text{t0, tf} = \text{initial and terminal values of } t \\ \text{y0} = \text{initial value of } y \text{ at } t_0 \end{cases}$$

C. For example, to numerically solve 
$$\begin{cases} t^2 y' = y + 3t \\ y(1) = -2 \end{cases}$$
 over the interval  $1 \leq t \leq 4$  :

- Create and save the file yp.m for the function  $\frac{1}{t^2}(y + 3t)$ .
- At a MATLAB prompt type:

```
[t,y]=ode45('yp',[1,4],-2);  
(your version of ode45 may not require brackets around [1,4])
```

- To print results type : `[t,y]`
- To plot results type : `plot(t,y)`
- To plot results type with a '+' symbol : `plot(t,y,'+')`

II. Second Order Equations 
$$\begin{cases} y'' + p(t)y' + q(t)y = g(t) \\ y(t_0) = y_0 \\ y'(t_0) = y_1 \end{cases}$$

A. First convert  $2^{nd}$  order equation above to an equivalent system of  $1^{st}$  order equations. Let  $x_1 = y$ ,  $x_2 = y'$  :

$$\begin{cases} x_1' = x_2 \\ x_2' = -q(t)x_1 - p(t)x_2 + g(t) \end{cases}, \text{ where } x_1(t_0) = y_0, x_2(t_0) = y_1.$$

B. Create and save a .m file which will return a vector-valued function. This is a little tricky so here is a specific example. Suppose the system is as below and  $0 \leq t \leq 4$

$$\begin{cases} x_1' = x_2 \\ x_2' = -t x_1 + e^t x_2 + 3 \sin 2t \end{cases}, \text{ where } x_1(0) = 2, x_2(0) = 8.$$

- Create the following function file and save it as **F.m** :

```
function xp=F(t,x)
xp=zeros(2,1); % since output must be a column vector
xp(1)=x(2);
xp(2)=-t*x(1)+exp(t)*x(2)+3*sin(2*t); % don't forget ; after each line
```

- Basic syntax for **ode45** . At MATLAB prompt, type :

```
[t,x]=ode45('F',[t0,tf],[x10,x20]);
```

$$\left\{ \begin{array}{l} \text{F= the .m file of the vector-function saved as above} \\ \text{t0, tf = initial and terminal values of } t \\ \text{x10 = initial value of } x_1 \text{ at } t_0 : \text{ x10} = x_1(t_0) \\ \text{x20 = initial value of } x_2 \text{ at } t_0 : \text{ x20} = x_2(t_0) \end{array} \right.$$

The example above becomes : `[t,x]=ode45('F',[0,4],[2,8]);`

- Since  $x_1(t) = y$ , to print out the values of the solution  $y$  for  $t_0 \leq t \leq t_f$ , type :

```
[t,x(:,1)]
```

To plot the solution on a graph  $t$  vs  $y$ , type : `plot(t,x(:,1))`

(This is because the vector  $\mathbf{x}$  has 1<sup>st</sup> component  $x_1 = y$  and 2<sup>nd</sup> component  $x_2 = y'$ .)

- To plot  $x_1$  vs  $x_2$  (phase plane) type : `plot(x(:,1),x(:,2))`