Extended Linear Response for Bioanalytical Applications Using Multiple Enzymes

Vladimir Privman, Oleksandr Zavalov, Aleksandr Simonian

\textsuperscript{a}Department of Physics, Clarkson University, Potsdam, NY 13699, USA
\textsuperscript{b}Materials Research and Education Center, Auburn University, Auburn, AL 36849, USA

*Corresponding author: phone (334) 844-4485, e-mail als@auburn.edu

\textbf{SUPPORTING INFORMATION}

Rate Equations

The set of the rate equations which correspond to Equations (3)-(8) is

\begin{align}
\frac{dE_1(t)}{dt} &= -k_1SE_1 + k_2C_1, \\ 
\frac{dS(t)}{dt} &= -k_1SE_1 - k_3SE_2 - k_4SE_2^a, \\ 
\frac{dE_2(t)}{dt} &= -k_3SE_2, \\ 
\frac{dE_2^a(t)}{dt} &= k_3SE_2 - k_4SE_2^a + k_5C_2, \\ 
\frac{dV(t)}{dt} &= \gamma(k_2C_1 + k_5C_2),
\end{align}

with all the concentrations on the right-hand sides time-dependent, and with $C_1(t) = E_1(0) - E_1(t)$ and $C_2(t) = E_2(0) - E_2(t) - E_2^a(t)$. 
Measure of the Linearity of the Response

The root-mean-square deviation of the slope of the response curve from the average slope is proportional to the following quantity,

\[ \sqrt{\int_0^{s_{\text{max}}} \left[ \frac{dP(S)}{dS} - \frac{P(s_{\text{max}}) - P(0)}{s_{\text{max}}} \right]^2 dS}, \]  

(S6)

where the average slope over the input range from 0 to \( s_{\text{max}} \) is

\[ \frac{|P(s_{\text{max}}) - P(0)|}{s_{\text{max}}}. \]  

(S7)

The measure \( \Delta \) can be defined as the room-mean-squared deviation normalized per the average slope,

\[ \Delta = \sqrt{\int_0^{s_{\text{max}}} \left[ \frac{dP(S)}{dS} - \frac{P(s_{\text{max}}) - P(0)}{s_{\text{max}}} \right]^2 dS \over \frac{|P(s_{\text{max}}) - P(0)|}{s_{\text{max}}}}. \]  

(S8)